The Hopf monoid of generalized permutahedra

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Joyal’s notion of species constitutes a good framework for the study of certain algebraic structures associated to combinatorial objects. We discuss the notion of Hopf monoid in the category of species and illustrate it with several examples. We introduce the Hopf monoid of generalized permutahedra (the latter are certain polytopes recently studied by Postnikov, Reiner and Williams). Our main result is an explicit antipode formula for this Hopf monoid. We explain how reciprocity theorems of Stanley on graphs and of Billera, Jia and Reiner on matroids can be deduced from this result. The talk is based on joint works with Swapneel Mahajan and with Federico Ardila.