Killing fields, mean curvature, translation maps

Susana Fornari, (UFMG, Belo Horizonte, Brazil)
Jaime Ripoll (UFRGS) Porto Alegre, Brazil)

D. Hoffman, R. Osserman and R. Schoen proved that if the Gauss map of a complete constant mean curvature (cmc) surface M immersed in R^3 is contained in a closed hemisphere of S^2(equivalently, the function $\langle \eta, v \rangle$ does not change sign on M where η is a unit normal vector of M and v some non zero vector in R^3), then M is invariant by a one parameter subgroup of translations of R^3 (the one determines by v). We obtain an extension of this result to the case that the ambient space is a Riemannian manifold N and M a hypersurface on N by requiring that the function $f := \langle \eta, V \rangle$ does not change sign on M , where V is a Killing field of N. A stability criterium for cmc surfaces in N^3 is also obtained. In the last part of the article we consider a Killing paralelizable Riemannian manifold N to define a translation map $\gamma : M \to R^n$ of a hypersurface M of N wich is a natural extension of the Gauss map of a hypersurface in R^n. Considering some hypothesis on the image of γ we obtain an extension to this setting of the original Hoffman-Osserman-Schoen result. This extension motivates to state in this context a conjecture made by M. P. do Carmo which, in R^3, reads: the Gauss image of a complete cmc surface which is not a plane nor a cylinder contains a neighborhood of some equator of the sphere.